University of Bahri
Journal of Arts and Human Sciences
مجلة جامعة بحري للآداب والعلوم الإنسانية

مجلة جامعة بحري للآداب والعلوم الإنسانية – المجلد 1، العدد 2، أغسطس 2026، الصفحات: 12 –19
University of Bahri Journal of Arts and Human Sciences (UBJAHS), Vol. 1, No. 2, August 2026
الناشر: جامعة بحري – عمادة البحث العلمي ردمد: 3122-6078
Imprint: University of Bahri -Deanship of Academic Research ISSN: 3122-6078

النمذجة العشوائية لدينميكيات السكان في السودان أثناء فترة النزاع المسلح الأخير

إنتصار الرشيد سيدأحمد محمد، أستاذ مشارك

جامعة بحري، كلية علوم الحاسوب والرياضيات، قسم الرياضيات

Email: entisar.alrasheed@bahri.edu.sd

⋆

المستخلص

في هذه الدراسة وبغرض توفير إطر نظري لفهم سلوك دينميكيات السكان في ظل النزاعات المسلحة قمنا بصياغة وايجاد حل لنموزج خطي عشوائي يصف ديناميكيات السكان خلال فترة النزاع المسلح الأخير.

تم تعريف فترة الحرب $[t_0, t_1]$ على أنها $t \in [t_0, t_1]$ حيث يمثل t_0 و t_1 بداية ونهاية فترة الحرب علي التوالي. قبل النزاع تم نمذجة ديناميكيات السكان بشكل عشوائي باستخدام الحركة البراونية الهندسية ثم تم إدخال حد الانحراف السلبي $\delta(t)$ المتغير مع الزمن، والذي يمثل الوفيات والنزوح والهجرة القسرية لوصف نمذجة ديناميكيات السكان أثناء النزاع.

وبتطبيق مبرهنة إيتو، تم اشتقاق المسار الزمني لحجم السكان، بالإضافة إلى القيمة المتوقعة والتباين خلال فترة النزاع.

أظهر التحليل بوضوح أن النزاع يؤدي إلى خفض معدل النمو الفعلي من r إلى $r-\delta$ مما يترتب عليه انخفاض الحجم المتوقع للسكان وزيادة مستوى عدم اليقين الديموغرافي الشئ الذي يتوافق مع التقارير المتعلقة بالأزمة الإنسانية المستمرة في السودان.

الكلمات المفتاحية: ديناميكيات السكان , النمذجة العشوائية,الحركة البراونية الهندسية, النزاع المسلح. السودان.

A Stochastic Model for Population Dynamics in Sudan During the Recent Armed Conflict

Entisar Alrasheed Sidahmed, Associate Professor

College of Computer Sciences and Mathematics, Department of Mathematics, University of Bahri, Sudan

Email: entisar.alrasheed@bahri.edu.sd

⋆

Abstract

In this study and in order to provide a theoretical framework to understand the behavior of the population dynamics under conflict we formulate and solve analytically the linear stochastic population model that describes the population dynamics in Sudan during the recent armed conflict. The war interval defined as $t \in [t_0, t_1]$ where t_0, t_1 represent the start and end times of the war period respectively.

Before conflict the population dynamics are modeled stochastically using Geometric Brownian Motion (GBM), The conflict period is then captured by introducing the time depending negative drift term to represent mortality, displacement and forced migration introduced in modelling of the population dynamics during the conflict. By applying Itô's lemma, we derived the population trajectory, expected value and the variance during the war.

The analysis shows clearly that the conflict reduces the effective growth rate from r to $r - \delta$, lowers the expected population size and increases demographic uncertainty which completely matched the reports on Sudan's ongoing humanitarian crisis.

Keywords: Stochastic Modeling, Stochastic Differential Equations, Geometric Brownian Motion, Armed Conflict, Sudan.

1 Introduction

Population dynamics usually influenced by both natural growth processes and external shocks such as conflicts, epidemics, pandemic and environmental changes. Traditional deterministic models provide a basic framework for population prediction but fail to capture the inherent randomness in growth rates and the uncertainty introduced by extreme events on other side we find that stochastic population models, which incorporate random fluctuations, offer a more realistic representation of population dynamics as they allow the assessment of variability and risk in population projections, providing valuable insight beyond the average behavior predicted by classical models.

This study makes three major contributions to the modeling of population dynamics under armed conflict. First, a conflict-dependent negative drift term is incorporated into a Geometric Brownian Motion (GBM) framework to capture the effects of war including excess mortality, displacement, and forced migration. Second, closed-form analytical expressions for the expected population size and variance during conflict periods are derived, allowing direct assessment of population decline and uncertainty. Third, an explicit extinction criterion is established to identify the threshold at which sustained conflict intensity leads to inevitable population collapse.

The article is organized as follows: Section 2 reviews the related literature. Section 3 presents the mathematical framework. Section 4 develops the pre-war population model. Section 5 introduces the stochastic population model under conflicts and its analytical solution. Section 6 analyzes the population dynamics during the war. Section 7 presents a comparison between the deterministic and stochastic models. Section 8 introduces the conclusion of the study. Section 9 includes some recommendation.

2 Literature Review

Generally, conflicts produce non-stationary population processes, where parameters such as growth rate and survival probability vary over time implies accruing of mortality, displacement and other demographic effects. Concerning the recent conflict in Sudan and according to the UNHCR recent reports [1] the war in Sudan that began in 2023 has created one of the largest displacement crises globally, and millions fleeing across borders. Oxfam reports [2] state that displacement in Sudan exceeded 10–14 million individuals.

Based on these reports clear that the population dynamics in Sudan strongly influenced by nonlinear shocks, including forced migration, mortality, that is the population trajectories in Sudan are no longer smooth or predictable but instead subject to large random fluctuations and discontinuities and clearly demonstrate how population systems deviate significantly from deterministic assumptions so, it's reasonable to use stochastic model in describing these dynamics this also align with the Demographic Transition Theory (DTT) [3] which suggests that war introduces structural breaks in population models, making classical deterministic approaches insufficient.

Using Stochastic Differential Equations (SDEs) in modelling population dynamics considered as a powerful framework for capturing the randomness and environmental fluctuations that deterministic models cannot adequately represent. While classical deterministic approaches, such as the logistic growth equation introduced by Verhulst (1838) [4], the predator–prey framework of Lotka (1925) [5] and Volterra (1926) [6], provided the foundational structure for population modelling, they fail to account for demographic noise, environmental variability, and sudden shocks such as conflict, displacement, and disease outbreaks. To address these limitations, Mao, Marion, and Renshaw (2002) [7] in their study Environmental Brownian Noise Suppresses Explosions in Population Dynamics, demonstrated that incorporating Brownian motion into Lotka–Volterra systems can stabilize population trajectories and prevent unrealistic explosions. Collectively, these contributions establish SDEs as an indispensable tool for modelling populations subjected to uncertainty, and they provide the methodological foundation for the present study, which adapts these stochastic frameworks to investigate population dynamics in Sudan during this recent conflict.

3 Mathematical Framework

Throughout this study we denote by $N(t)$ to the population size at time t and consider the one-dimensional adapted standard Brownian motion $W(t)$ defined on the complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in T}, P)$.

To ensure the existence and uniqueness solution to the stochastic differential equation that describe the population dynamics during the conflict (introduced bellow) we will put the following assumptions [8]

1. Initially $N(0) = N_0 > 0$
2. The intrinsic growth rate r and the volatility σ are positive constants.
3. The population is considered without classification by age, gender or spatial structure.

4 Pre-War Population Model

The stochastic linear population growth model follows the Geometric Brownian Motion in the form [8]

$$dN(t) = rN(t)dt + \sigma N(t)dW(t) \tag{1}$$

known as standard stochastic population growth model in the absence of conflicts so, we will deal with it as reference model representing the natural demographic trajectory and population dynamics of Sudan in the absence of armed conflict.

The term $rN(t)dt$ is the natural population growth and $\sigma N(t)dW(t)$ is the stochastic fluctuations. Applying the standard stochastic calculus to the transformation $Y(t) = \ln N(t)$ and denoting the population size at the initial time by N_0 then, equation (1) has general solution in the form [8]

$$N(t) = N_0 \exp\left[\left(r - \frac{\sigma^2}{2}\right)t + \sigma W(t)\right] \tag{2}$$

Which is log-normally distributed random variable with
Expected value $E[N(t)] = N_0 e^{rt}$
Variance $Var[N(t)] = N_0^2 e^{2rt}(e^{\sigma^2 t} - 1)$.

5 Linear Stochastic Population Model Under War Effect

The negative impact on the population due to the war considered as an external shock represented as an extra term added to the model (1), this term is time-dependent negative drift function that decreases the population at a rate proportional to its current size, so the linear stochastic population model during war can be defined as:

$$dN(t) = (r - \delta(t))N(t)dt + \sigma N(t)dW(t) \tag{3}$$

Here $\delta(t)$ represents the intensity of population loss rate due to war during time interval $[t_0, t_1]$ and $\delta(t)N(t)$ is the term that measuring the expected decrease in population caused by war.

Based on the war time interval $[t_0, t_1]$ we will define the war intensity function $\delta(t)$ as piece-wise function in the form:

$$\delta(t) = \begin{cases} 0, & t < t_0 \\ \delta, & t_0 \le t \le t_1 \\ 0, & t > t_1 \end{cases} \tag{4}$$

Where δ is a constant representing the proportional reduction in population due to war attributable to factors such as excess deaths, migration, and forced displacement, etc. During the war the drift changes from r to $r - \delta$.

Equation (3) can be written the form:

$$dN(t) = (r - \delta)N(t)dt + \sigma N(t)dW(t) \tag{5}$$

Equation (5) is SDE with time-dependent drift, this equation serves as the general form that describes the population dynamics during war effect. Applying Itô's lemma to (5) has a general solution of the form [8]

$$N(t) = N(t_0) \exp\left[\left(r - \delta - \frac{\sigma^2}{2}\right)(t - t_0) + \sigma(W(t) - W(t_0))\right] \tag{6}$$

6 Population Dynamics During the War

Based on the definition of $\delta(t)$ in (4), then during the war interval $[t_0, t_1]$ the population model takes the form:

$$dN(t) = (r - \delta)N(t)dt + \sigma N(t)dW(t) \tag{7}$$

The terms in Equation (7) interpreted as follow:
$N(t_0)$ is the population size at the onset of the conflict
$(r-\delta)$, represents the effective deterministic growth during the war.
$\sigma dW(t)$, represents the random disturbances related to war.

6.1 Expected Population Size During the War

Taking the expectation of (7) gives

$$E[N(t)] = N_0 e^{(r-\delta)t} \tag{8}$$

From (8) it's clear that the war lowers the expected population at a constant value where the magnitude of this decline depends on the time difference, therefore the expected population at any moment during the war is strictly smaller than what it would have been in the absence of conflict.

6.2 Variance of the Population During the War

During the war interval the variance of the population satisfies

$$Var[N(t)] = N_0^2 e^{2(r-\delta)t}(e^{\sigma^2 t} - 1) \tag{9}$$

Variance in equation (9) reflects how widely population outcomes can deviate from the normal expected path due to random mortality, displacement, force migration shocks, and unpredictable war events. Since due to the war the population size will be unstable so, σ^2 will be larger, and therefore the term becomes much larger, causing a large variance, this high variance during war means that the future population size becomes unpredictable, there will be large dispersion around the average population.

6.3 Almost Sure Extinction Condition

By taking the logarithm of equation (7) and dividing both sides by t, letting $t \to \infty$, and using the strong law of large numbers for Brownian motion [9], we obtain

$$\lim_{t\to\infty} \frac{\ln N(t)}{t} = r - \delta - \frac{\sigma^2}{2} \tag{10}$$

So, $N(t) \to 0$ if and only if $r - \delta - \frac{\sigma^2}{2} < 0$ or

$$\delta > r - \frac{\sigma^2}{2} \tag{11}$$

The condition (11) shows that if the war intensity exceeds the effective growth rate, the population is asymptotically driven toward extinction.

7 Comparison with the Deterministic Model

In the absence of conflicts and when no randomness at all, the population changes in a completely predictable way so, by setting $\sigma = 0$ in equation (3) the model will take the form

$$\frac{dN(t)}{dt} = rN(t) \tag{12}$$

with general solution in the form

$$N(t) = N_0 e^{rt} \tag{13}$$

(13) says that the population size grows if $r > 0$ and declines if $r < 0$.

Comparison of equations (12) and (8) indicates that the deterministic solution coincides with the expected value of the stochastic model. However, the deterministic formulation yields only a single point estimate, whereas the stochastic formulation also captures demographic risk through the variance term given in (9).

8 Conclusion

This study used a stochastic linear population model to describe the population dynamics of Sudan during the recent armed conflict. The model demonstrates that conflict introduces an additional mortality term which strongly reduces the natural growth rate from r to $r - \delta$, leading to a significant decline in the expected population during the conflict period. The analytical solution provides expected value, variance, and identifies the condition which the population is driven toward almost-sure extinction if the war continues.

9 Recommendation

The related coming studies should emphasize on

1. The use of stochastic models in demographic analysis under armed conflicts or generally uncertainty.
2. If possible, integrate the theoretical study with real-world data.
3. Extending the model to a multi-compartment formulation distinguishing civilians, internally displaced persons, refugees, age and gender.
4. Encouraging policymakers to consider both expected outcomes and variability when planning in conflict-affected regions.

References

(1) UNHCR (2024–2026). Sudan Emergency Reports. https://www.unhcr.org/countries/sudan

(2) Oxfam (2026). Sudan Displacement Crisis Report. https://www.oxfam.org

(3) Verhulst, P. F. (1838). Notice sur la loi que la population suit dans son accroissement. *Correspondance Mathématique et Physique*, 10, 113–121.

(4) Lotka, A. J. (1925). *Elements of Physical Biology*. Williams & Wilkins, Baltimore.

(5) Volterra, V. (1926). Variazioni e fluttuazioni del numero d'individui in specie animali conviventi. *Memorie della R. Accademia Nazionale dei Lincei*, 2, 31–113.

(6) Mao, X., Marion, G., & Renshaw, E. (2002). Environmental Brownian noise suppresses explosions in population dynamics. *Stochastic Processes and their Applications*, 97(1), 95–110.

(7) Øksendal, B. (2003). *Stochastic Differential Equations: An Introduction with Applications*. Springer.

(8) Mörters, P. and Peres, Y. (2010). *Brownian Motion*. Cambridge University Press.

(9) Allen, L. J. S. (2011). *An Introduction to Stochastic Processes with Applications to Biology*. CRC Press.

(10) Bao, J., & Yuan, C. (2012). Stochastic population dynamics driven by Lévy noise. *Journal of Mathematical Analysis and Applications*, 391(2), 363–375.

(11) Caraballo, T., Colucci, R., López-de-la-Cruz, J., & Rapaport, A. (2024). A way to model stochastic perturbations in population dynamics models with bounded realisations. *Communications in Nonlinear Science and Numerical Simulation*.